University of Belgrade
Faculty of Organizational Sciences

27 June 2017

MATHEMATICS ENTRANCE EXAM

The test contains 20 questions on 2 pages. Each question is worth 5 points. If you do not wish to choose one of the first five offered answers, you may mark "N", which is worth 0 points. For an incorrect answer, 0.5 points are deducted. If, for a given question, more than one answer is marked or no answer is marked, as well as if the answer is marked incorrectly in any way, 1 point is deducted.

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1. The value of the expression $\dfrac{(2+\sqrt{3})\cdot 64^{(-2)^{-2}}\cdot \sqrt[3]{(-1)^3}}{(2-\sqrt{3})^{-1}\cdot 64^{-2^{-2}}\cdot \sqrt[4]{(-8)^4}}$ is:

A) 1; B) 4; C) 8; D) $2+\sqrt{3}$; (E) -1; N) I don't know.

2. If the length of one edge of a cuboid is increased by 20%, the length of another edge is decreased by 20%, and the length of the third edge remains unchanged, then the volume of the cube:

A) does not change; B) increases by 5%; (C) decreases by 4%;
D) increases by 4%; E) decreases by 10%; N) I don't know.

3. The value of the expression $2^{\log_{0.25}\left(\log_4 2^{\frac{2}{3}}\right)}$ is equal to:

A) $\dfrac{1}{3}$; B) 1; C) 3; (D) $\sqrt{3}$; E) $\sqrt[3]{2}$; N) I don't know.

4. If a, b and c are real numbers such that $b > a$ and $a+b \neq c$, then the expression $\dfrac{\sqrt{(a-b)^2}}{\sqrt[3]{(a-b)^3}}\cdot\dfrac{a^2-b^2-c^2+2bc}{a+b-c}$ is identically equal to:

A) $a+b+c$; B) $(a-b)(a-b-c)$; (C) $-a+b-c$; D) $\dfrac{b+c-a}{a-b}$; E) $a-b+c$; N) I don't know.

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5. Let a, b and c be the values for which the polynomial $P(x) = x^{2017} + ax^{2014} + bx^{1001} + c$ is divisible by x^2+1, and when divided by $x-1$ it gives remainder 4. Then $a^3+b^3+c^3$ is equal to:

(A) 15; B) 12; C) 9; D) 17; E) 3; N) I don't know.

6. If n is a natural number, $f(n) = \left(\dfrac{1+i}{\sqrt{2}}\right)^n + \left(\dfrac{1-i}{\sqrt{2}}\right)^n$ and $i^2 = -1$, then $f(2017) + f(2013)$ is:

A) $\sqrt{2}$; (B) 0; C) $4i$; D) $-2\sqrt{2}$; E) $2\sqrt{2}$; N) I don't know.

7. Consider the real functions $f_1(x) = \log_3(x^2-10x+21)$, $f_2(x) = \dfrac{1}{\sqrt[4]{10x-x^2}}$ and $f_3(x) = \dfrac{\log_4(x^2+3)}{\sqrt{4-x}}$. If D_{f_1}, D_{f_2} and D_{f_3} are the domains of the functions f_1, f_2 and f_3 respectively, then the correct statement is:

A) $D_{f_1}\cup D_{f_2}\cup D_{f_3} = (-\infty, 7)$; (B) $D_{f_1}\cap D_{f_2}\cap D_{f_3} = (0,3)$; C) $(D_{f_1}\cup D_{f_2})\cap D_{f_3} = (0,4)$;
D) $D_{f_1}\cap D_{f_2}\cap D_{f_3} = (0,4)$; E) $(D_{f_1}\cup D_{f_2})\cap D_{f_3} = (-\infty, 4]$; N) I don't know.

8. The product of all real solutions of the equation $\log_2(x+4) = \log_{4x+16} 8$ is equal to:

A) $\dfrac{31}{2}$; B) 15; C) $-\dfrac{31}{2}$; D) -15; (E) $\dfrac{31}{4}$; N) I don't know.

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9. If $a \in (-\infty, +\infty) \setminus \left\{-\frac{1}{2}\right\}$, then the solutions of the quadratic equation $x^2 - (a+2)x + 2a + 1 = 0$ are distinct and of the same sign if and only if:

A) $a \in \left(-\infty, -\frac{1}{2}\right) \cup \left(-\frac{1}{2}, +\infty\right)$; B) $a \in \left(-\frac{1}{2}, +\infty\right)$; (C) $a \in \left(-\frac{1}{2}, 0\right) \cup (4, +\infty)$;
D) $a \in \left(-\frac{1}{2}, 0\right) \cup [4, +\infty)$; E) $a \in \left(-\infty, -\frac{1}{2}\right) \cup (4, +\infty)$; N) I don't know.

10. The set of all real solutions of the inequality $\sqrt{4 - 4^x} > 2 - 2^x$ is:

A) $(-\infty, 1]$; B) $[-2, 1)$; C) $[0, 1)$; (D) $(-\infty, 1)$; E) $[-1, 1)$; N) I don't know.

11. The product of all real solutions of the equation $2x^2\sqrt{1-x^2} + 4\sqrt{1-x^2} = 9x\sqrt{1-x^2}$ is equal to:

A) 1; B) 2; C) $\frac{1}{2}$; (D) $-\frac{1}{2}$; E) -2; N) I don't know.

12. In a triangle, one interior angle is equal to the difference of the other two interior angles. The ratio of the two shorter sides is $3 : 4$. If the area of the triangle is $24\,\mathrm{cm}^2$, the circumference of the circumscribed circle of that triangle is equal to:

A) 5π cm; B) 6π cm; C) 3π cm; D) 7π cm; (E) 10π cm; N) I don't know.

13. Out of 12 books, 4 are about mathematics. The number of different ways to purchase 3 books such that at least one is about mathematics is:

A) 24; (B) 164; C) 56; D) 984; E) 220; N) I don't know.

14. A line p containing one focus of the hyperbola $4x^2 - 5y^2 = 20$ and perpendicular to the x-axis intersects the hyperbola at points A and B. The perimeter of the triangle whose vertices are points A, B, and the focus of the hyperbola that does not belong to line p is:

(A) $\frac{36}{\sqrt{5}}$; B) $\frac{20}{\sqrt{5}}$; C) $\frac{40}{\sqrt{5}}$; D) $\frac{28}{\sqrt{5}}$; E) 18; N) I don't know.

15. If $a_1, a_2, a_3, \ldots$ is a decreasing geometric progression the sum of whose first three terms is 28, and if $a_1, a_2, a_3 - 4$ are the first three terms of some arithmetic progression, then the sum of the first four terms of that arithmetic progression is equal to:

A) 28; (B) 16; C) -4; D) 24; E) 32; N) I don't know.

16. The value of the expression $\sin 54^\circ \cos 108^\circ$ is equal to:

A) $-\frac{1}{8}$; B) $-\frac{\sqrt{3}}{4}$; C) $-\frac{1}{2}$; D) $-\frac{\sqrt{2}}{4}$; (E) $-\frac{1}{4}$; N) I don't know.

17. If the sum of all binomial coefficients in the expansion of $\left(\frac{1}{x} + 2x\right)^n$, $x \neq 0$, is equal to 2^{12}, then the term that does not depend on x is:

A) 462; B) 924; C) 64; D) $2^7 \cdot 231$; (E) $2^8 \cdot 231$; N) I don't know.

18. A regular triangular prism with a volume of $54\ \mathrm{cm}^3$ has the minimum sum of the lengths of all edges if the side length of its base is equal to:

(A) $\frac{6}{\sqrt[6]{3}}$ cm; B) $\frac{36}{\sqrt[6]{2}}$ cm; C) $\frac{42}{\sqrt[6]{3}}$ cm; D) $\frac{\sqrt[6]{3}}{2}$ cm; E) $\frac{\sqrt[6]{2}}{3}$ cm; N) I don't know.

19. The lengths of the bases of a trapezoid are 20 cm and 6 cm, and the legs are 13 cm and 15 cm long. By rotating the trapezoid around the longer base, a solid is formed whose volume is equal to:

A) $1440\pi\ \mathrm{cm}^3$; B) $1560\pi\ \mathrm{cm}^3$; C) $1600\pi\ \mathrm{cm}^3$; (D) $1536\pi\ \mathrm{cm}^3$; E) $1920\pi\ \mathrm{cm}^3$; N) I don't know.

20. The sum of all solutions of the equation $\sin x + \sin 2x + 1 = \cos x + 2\cos^2 x$ belonging to the interval $(-\pi, \pi)$ is equal to:

A) $\frac{\pi}{3}$; (B) $\frac{\pi}{2}$; C) $\frac{3\pi}{2}$; D) $-\pi$; E) $-\frac{\pi}{2}$; N) I don't know.