

## MATHEMATICS ENTRANCE EXAM

The test contains 20 questions on 2 pages. Each question is worth 5 points. If you do not wish to choose one of the first five offered answers, you may mark "N", which is worth 0 points. For an incorrect answer, 0.5 points are deducted. If, for a given question, more than one answer is marked or no answer is marked, as well as if the answer is marked incorrectly in any way, 1 point is deducted.

1. The expression  $\frac{2a + (a^2 - 1)^{\frac{1}{2}}}{((a - 1)^{\frac{1}{2}} + (a + 1)^{\frac{1}{2}}) \cdot ((a - 1)^{\frac{3}{2}} - (a + 1)^{\frac{3}{2}})}$ , where  $a$  is a real number and  $a \geq 1$ , is identically equal to:

A)  $\frac{1}{2a}$ ;      B)  $\frac{2a}{2a - (a^2 - 1)^{\frac{1}{2}}}$ ;      C)  $\frac{2a}{a + (a^2 - 1)^{\frac{1}{2}}}$ ;       D)  $-\frac{1}{2}$ ;      E) 2;      N) I don't know.

2. In  $K$  liters of a water-salt solution, there is 80% salt. If by adding 10 liters of water to this solution we obtain a solution that contains 60% salt, then  $K$  is equal to:

A) 30;      B) 35;      C) 20;      D) 12;      E) 45;      N) I don't know.

3. The value of the expression  $\frac{\sqrt[3]{12 + 4\sqrt{5}} \cdot \sqrt[3]{12 - 4\sqrt{5}}}{\sqrt[6]{4 - 2\sqrt{3}} \cdot \sqrt[3]{4(1 + \sqrt{3})}}$  is:

A)  $-1$ ;      B) 1;      C) 4;       D) 2;      E)  $-2$ ;      N) I don't know.

4. If the complex number  $z$  satisfies the equation  $\bar{z}(4 - 2i) + 3\sqrt{3} + 3i(1 + 2\sqrt{3} + 2i) = 0$ , where  $i$  is the imaginary unit ( $i^2 = -1$ ), then  $\operatorname{Re}(z) \cdot \operatorname{Im}(z)$  is equal to:

A)  $\frac{9\sqrt{3}}{4}$ ;      B)  $\frac{25\sqrt{3}}{4}$ ;      C)  $-\frac{9\sqrt{3}}{4}$ ;      D)  $-\frac{25\sqrt{3}}{4}$ ;      E)  $\frac{81\sqrt{3}}{4}$ ;      N) I don't know.

5. If the remainder when dividing the polynomial  $x^{2018} - ax^{2017} + bx^{2015} + c$  by the polynomial  $x^3 - x^2 + x - 1$  is equal to  $ax + b$ , where  $a, b, c$  are real numbers, then the value of  $a - b + c$  is equal to:

A)  $\frac{5}{2}$ ;      B) 0;       C)  $\frac{3}{2}$ ;      D) 1;      E)  $-\frac{3}{2}$ ;      N) I don't know.

6. The intersection of the natural domains of the real functions  $f_1(x) = \sqrt{\frac{\log_2(x+1)}{x-1}}$ ,  $f_2(x) = \sqrt{\frac{x^2 + 3x + 4}{x-3}}$  and  $f_3(x) = \log_7(4 + 5x) + \sqrt{7^{2x} - 2401}$  is:

A)  $(-1, 0] \cup (1, 3) \cup (3, 4)$ ;      B)  $[2, 3) \cup (3, +\infty)$ ;      C)  $\left(-\frac{4}{5}, 1\right) \cup (1, 3) \cup (3, 7)$ ;  
D)  $(1, 3) \cup (3, +\infty)$ ;       E)  $(3, +\infty)$ ;      N) I don't know.

7. The number of all real solutions of the equation  $\log_2(\log_3 x) + \log_2(\log_3 x^2 - 3) = 1$  is:

A) 2;      B) 3;       C) 1;      D) 4;      E) 0;      N) I don't know.

8. The value of the expression  $\log_4 \log_3 \log_2 8 + \log_{\sqrt{7}+1}(8 + 2\sqrt{7}) \cdot \log_{\sqrt[3]{7}} 7\sqrt{7}$  is:

A) 9;      B) 11;      C)  $\frac{9}{2}$ ;      D) 3;      E) 10;      N) I don't know.

9. The number of all integer solutions of the inequality  $5^{2 - \left(\frac{1}{3}\right)^{\frac{x}{2}}} < 0.2$  is:  
 A) 2;                      B) 3;                      C) 0;                      **(D)** 1;                      E) 4;                      N) I don't know.
10. The solutions of the equation  $(m + 1)x^2 + mx - m + 1 = 0$ , where  $m$  is a real number and  $m \neq -1$ , are negative and distinct if and only if  $m$  belongs to the set:  
 A)  $\left[-\frac{2}{\sqrt{5}}, 1\right)$ ;    B)  $\left[\frac{2}{\sqrt{5}}, 1\right)$ ;    C)  $(-1, 1)$ ;    D)  $(0, 1)$ ;    **(E)**  $\left(\frac{2}{\sqrt{5}}, 1\right)$ ;    N) I don't know.
11. The number of all negative integer solutions of the inequality  $(x + 3)\sqrt{12 - |x|} \geq 0$  is equal to:  
 A) 3;                      **(B)** 4;                      C) 12;                      D) 5;                      E) 6;                      N) I don't know.
12. In a trapezoid with area  $\frac{3}{2}(11 - \sqrt{3})\text{ cm}^2$ , the longer base is 7 cm and the height is 3 cm. If one of the base angles is  $45^\circ$ , then the perimeter of the trapezoid (in cm) is equal to:  
 A) 18;    **(B)**  $11 + \sqrt{3} + 3\sqrt{2}$ ;    C)  $11 + 3\sqrt{2}$ ;    D) 20;    E)  $11 + 3\sqrt{3}$ ;    N) I don't know.
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13. The real numbers  $a$ ,  $b$  and  $c$ , whose sum is equal to 19, form an increasing geometric progression. If the numbers  $a$ ,  $b$  and  $c - 1$  form an arithmetic progression, then  $c \cdot (a + b)^{-1}$  is equal to:  
 A)  $\frac{4}{15}$ ;    **(B)**  $\frac{9}{10}$ ;    C) 90;    D)  $\frac{15}{4}$ ;    E)  $\frac{10}{9}$ ;    N) I don't know.
14. The value of the expression  $\log_2 \cos 20^\circ + \log_2 \cos 40^\circ + \log_2 \cos 80^\circ$  is equal to:  
**(A)**  $-3$ ;    B)  $-\sqrt{2}$ ;    C) 2;    D) 3;    E)  $-2$ ;    N) I don't know.
15. The center  $C(p, q)$  of a circle with radius  $r$  lies on the line  $5x - 3y + 12 = 0$ . If the circle contains the points  $A(1, -3)$  and  $B(1, 1)$ , then the value of  $p + q + r^2$  is equal to:  
 A) 20;    **(B)** 16;    C) 18;    D) 24;    E) 17;    N) I don't know.
16. A hemisphere is inscribed in a right cone of volume  $12\pi\text{ cm}^3$  such that the hemisphere touches the lateral surface of the cone along a circle, and the base of the hemisphere lies on the base of the cone. If the height of the cone is 4 cm, then the volume of the hemisphere is equal to:  
 A)  $\frac{384\pi}{125}\text{ cm}^3$ ;    B)  $\frac{2304\pi}{125}\text{ cm}^3$ ;    C)  $\frac{576\pi}{125}\text{ cm}^3$ ;    **(D)**  $\frac{1152\pi}{125}\text{ cm}^3$ ;    E)  $\frac{128\pi}{125}\text{ cm}^3$ ;    N) I don't know.
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17. The sum of all solutions of the equation  $3(1 - \sin x) + \sin^4 x = 1 + \cos^4 x$  that belong to the interval  $(-\pi, \pi)$  is equal to:  
 A)  $\frac{2\pi}{3}$ ;    B)  $2\pi$ ;    **(C)**  $\frac{3\pi}{2}$ ;    D)  $\frac{4\pi}{3}$ ;    E)  $\pi$ ;    N) I don't know.
18. A right regular triangular prism with volume  $4\text{ cm}^3$  is circumscribed by a cylinder such that their bases lie in the same planes. The minimum surface area of such a circumscribed cylinder is:  
 A)  $2\sqrt[3]{4}\pi\text{ cm}^2$ ;    B)  $4\pi\text{ cm}^2$ ;    C)  $16\pi\text{ cm}^2$ ;    **(D)**  $8\pi\text{ cm}^2$ ;    E)  $4\sqrt[3]{4}\pi\text{ cm}^2$ ;    N) I don't know.
19. The number of ways to choose three natural numbers less than 31 such that they do not all have the same remainder upon division by 3 is equal to:  
 A) 3940;    B) 3820;    **(C)** 3700;    D) 5700;    E) 1000;    N) I don't know.
20. In the expansion of  $(3 - \sqrt[3]{5})^n$  the sum of all binomial coefficients is equal to  $4^{1009}$ . The number of all terms in this expansion that are negative integers is equal to:  
 A) 673;    B) 0;    C) 672;    D) 337;    **(E)** 336;    N) I don't know.
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