University of Belgrade
Faculty of Organizational Sciences

25 June 2019

MATHEMATICS ENTRANCE EXAM

The test contains 20 questions on 2 pages. Each question is worth 5 points. If you do not wish to choose one of the first five offered answers, you may mark "N", which is worth 0 points. For an incorrect answer, 0.5 points are deducted. If, for a given question, more than one answer is marked or no answer is marked, as well as if the answer is marked incorrectly in any way, 1 point is deducted.

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1. Let a and b be positive real numbers. If increasing a by 50% and increasing b by p% increases their product by 80%, then p is equal to:

(A) 20; B) 120; C) 30; D) 25; E) 40; N) I don't know.

2. Given the complex number $z = \left(\frac{2+4i}{-1+3i}\right)^{2019}$, where i is the imaginary unit ($i^2 = -1$), the sum $\mathrm{Re}(z) + \mathrm{Im}(z)$ is equal to:

A) 0; B) -2^{1009}; C) 2^{1009}; (D) -2^{1010}; E) 2^{1010}; N) I don't know.

3. The value of the expression $\frac{8^{\frac{-1}{3}}}{\sqrt[4]{0.0081}\cdot 625^{0.75}} + \frac{2^5\cdot 5^{-2}\cdot(5-(-2))}{3}$ is equal to:

A) $\frac{161}{75}$; (B) 3; C) $\frac{1}{3}$; D) 1; E) $\frac{228}{75}$; N) I don't know.

4. The expression $\frac{(a+b)^3-(a-b)^3}{(a+b)^2-(a-b)^2} : \frac{2b(3a^2+b^2)}{(\sqrt{a}+\sqrt{b})^2-(\sqrt{a}-\sqrt{b})^2}$ where a and b are positive real numbers, is identically equal to:

A) $\frac{3a^2+b^2}{\sqrt{a}+\sqrt{b}}$; B) $\frac{1}{ab}$; (C) $\frac{1}{\sqrt{ab}}$; D) $\sqrt{ab}$; E) $\frac{2ab}{(\sqrt{a}+\sqrt{b})^2}$; N) I don't know.

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5. The sum of all real solutions of the equation $1+\log_2\frac{x+1}{x+2} = \frac{1}{4}\log_{\sqrt{2}}(x-2)^2$ is:

A) 2; (B) $\sqrt{7}-1$; C) 0; D) $\sqrt{7}-\sqrt{3}$; E) -2; N) I don't know.

6. If $a = \log_3 4\cdot\log_4 5\cdot\log_5 6$ and $b = \frac{\log_2 81}{1+\log_2 3}$, then the value of $\log_2 ab$ is equal to:

A) $\log_2 6$; B) 4; C) $\log_2 9$; D) 1; (E) 2; N) I don't know.

7. If the remainder when dividing the polynomial $x^4 - ax^3 - 3ax$ by the polynomial x^2-4x+4 is equal to $ax+2b$, where a and b are real numbers, then the value of $a+b$ is equal to:

A) 10; B) 0; C) -4; D) 22; (E) -6; N) I don't know.

8. If $g(x-2) = 2x+1$, $g(f(x)+2) = 2x-15$, and $h(x) = f(g(x))$, then:

(A) $h(x) = 2x-7$; B) $h(x) = 2x-3$; C) $h(x) = 2x-10$; D) $h(x) = 2x-12$; E) $h(x) = 2x-5$; N) I don't know.

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9. The product of all integer solutions of the equation $\sqrt[3]{x+5}+\sqrt[3]{x+6}=\sqrt[3]{2x+11}$ is:

(A) 30; B) 165; C) -30; D) -165; E) -11; N) I don't know.

10. The product of all real solutions of the equation $x^2+x+\frac{2}{x}+\frac{4}{x^2}=2$ is:

(A) 2; B) 8; C) 1; D) 6; E) 4; N) I don't know.

11. If the vertices of an isosceles trapezoid $ABCD$ with bases AB and CD lie on a circle with center O and radius 1 cm, and if $\measuredangle AOD = 30^\circ$ and $\measuredangle DOC = 60^\circ$, then the area of the trapezoid in cm^2 is equal to:

A) $1+\frac{\sqrt{3}}{2}$; (B) $\frac{1}{2}$; C) 1; D) $\frac{\sqrt{3}}{2}$; E) $\frac{\sqrt{3}+1}{2\sqrt{2}}$; N) I don't know.

12. Consider the equation $2^{3\sqrt{x^2+2x}}+3\cdot 2^{-\sqrt{x^2+2x}}=2^{-3\sqrt{x^2+2x}}+3\cdot 2^{\sqrt{x^2+2x}}$. The sum of the cubes of all integer solutions of this equation is equal to:

A) 35; B) 19; C) 8; (D) -8; E) 0; N) I don't know.

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13. A right circular cylinder is inscribed in a right frustum of a cone with volume $3150\pi\,\mathrm{cm}^3$ and base diameters 40 cm and 10 cm, such that one base of the cylinder lies on the larger base of the frustum, and the other base touches the lateral surface of the frustum along its entire circumference. If the height of the cylinder is equal to the diameter of its base, the surface area of the cylinder is equal to:

A) $\frac{225\pi}{2}\,\mathrm{cm}^2$; B) $\frac{86400\pi}{121}\,\mathrm{cm}^2$; C) $225\pi\,\mathrm{cm}^2$; D) $\frac{2025\pi}{2}\,\mathrm{cm}^2$; (E) $\frac{675\pi}{2}\,\mathrm{cm}^2$; N) I don't know.

14. Let A be the orthogonal projection of the point $B(16,-1)$ onto the line $y-5x+3=0$. The product of the distances from point A to the foci of the ellipse $9x^2+25y^2=225$ is equal to:

A) $4\sqrt{2}$; B) $\sqrt{29}$; C) 10; (D) $\sqrt{377}$; E) 13; N) I don't know.

15. The value of the expression $\sin 975^\circ+\cos 975^\circ$ is equal to:

A) $-\frac{\sqrt{3}}{2}$; B) $-\frac{\sqrt{2}}{2}$; C) $-\frac{1}{2}$; D) $-\frac{\sqrt{6}}{4}$; (E) $-\frac{\sqrt{6}}{2}$; N) I don't know.

16. If the sum of three numbers that are consecutive terms of an increasing geometric progression is 42, and the sum of their reciprocals is $\frac{21}{32}$, then their product is equal to:

A) 256; B) 64; (C) 512; D) $\frac{441}{16}$; E) 216; N) I don't know.

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17. The maximum volume of a regular square pyramid whose lateral edge has length 3 cm is equal to:

A) $3\sqrt{3}\,\mathrm{cm}^3$; B) $12\,\mathrm{cm}^3$; C) $6\sqrt{3}\,\mathrm{cm}^3$; D) $4\sqrt{2}\,\mathrm{cm}^3$; (E) $4\sqrt{3}\,\mathrm{cm}^3$; N) I don't know.

18. In the expansion $(\sqrt[3]{3}-\sqrt{2})^{2019}$, the number of all terms that are natural numbers is equal to:

A) 336; B) 673; C) 168; (D) 337; E) 169; N) I don't know.

19. The number of solutions of the equation $2\cos x\cos 2x=\cos x-\frac{1}{2}$ that belong to the interval $\left[-\frac{8\pi}{9},\frac{8\pi}{9}\right)$ is equal to:

A) 3; B) 2; (C) 5; D) 4; E) 1; N) I don't know.

20. The number of all odd six-digit numbers that contain at least one and at most three zeros is equal to:

A) 211680; B) $70\cdot 9^4$; C) $14\cdot 5^5$; (D) $1910\cdot 9^2$; E) $2\cdot 5^6$; N) I don't know.