University of Belgrade
Faculty of Organizational Sciences

30 June 2020

MATHEMATICS ENTRANCE EXAM

The test contains 20 questions on 2 pages. Each question is worth 5 points. If you do not wish to choose one of the first five offered answers, you may mark "N", which is worth 0 points. For an incorrect answer, 0.5 points are deducted. If, for a given question, more than one answer is marked or no answer is marked, as well as if the answer is marked incorrectly in any way, 1 point is deducted.

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1. The value of the expression $\dfrac{\left(\sqrt{3^4}+\sqrt[3]{3^6}\right)\cdot 3^{-2}+3^0}{\sqrt[5]{(-3)^5}-\sqrt[4]{(-3)^4}}$ is equal to:

A) 1; B) -1; C) $-\frac{1}{3}$; D) $\frac{1}{2}$; (E) $-\frac{1}{2}$; N) I don't know.

2. For $a\neq 0$, $b\neq 0$ and $a\neq -b$, the expression $\left(\dfrac{a^{-1}}{b^{-1}}+\dfrac{b^{-2}}{a^{-2}}\right)^{-1}\cdot\left(\dfrac{a}{b}+\dfrac{b}{a}-1\right)+\dfrac{1}{b}:\left(\dfrac{1}{a}+\dfrac{1}{b}\right)$ is identically equal to:

(A) 1; B) $\frac{b}{a+b}$; C) $\frac{b-a}{a+b}$; D) 0; E) $\frac{a}{a+b}$; N) I don't know.

3. Let $f\left(\dfrac{x}{1-x}\right)=\dfrac{x}{2}$ for $x\neq 1$ and $g(x)=f(x)+f(-x)$ for $x\neq\pm 1$. Then:

A) $g(x)=\frac{x^2+1}{x^2-1}$; B) $g(x)=\frac{x^2}{1-x^2}$; C) $g(x)=\frac{2x}{1-x^2}$; D) $g(x)=\frac{2x}{x^2-1}$; (E) $g(x)=\frac{x^2}{x^2-1}$; N) I don't know.

4. If $|z|+\bar{z}=3+i\sqrt{3}$, $i^2=-1$, then $(z-1)^{2020}$ is equal to:

A) -3^{1010}; (B) 3^{1010}; C) $3^{2020}i$; D) 3^{2020}; E) $-3^{1010}i$; N) I don't know.

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5. The sum of three natural numbers is equal to 1995. If the first number is 30% greater than the second, and the second is 30% greater than the third, then the second number is equal to:

A) 605; B) 635; (C) 650; D) 665; E) 620; N) I don't know.

6. The number of all integer solutions of the inequality $\dfrac{x^2+2x-15}{x^2+5x+6}\leq -1$ is equal to:

A) 2; (B) 4; C) 0; D) 1; E) 3; N) I don't know.

7. The remainder obtained by dividing the polynomial $P(x)=x^{2020}+2x^{2019}-1$ by the polynomial $Q(x)=x^3+x^2+x+1$ is equal to:

A) $2x^2-2x-2$; B) $2x^2+2x+2$; C) x^2+2x+1; (D) $-2x^2-2x-2$; E) $-x^2-2x-1$; N) I don't know.

8. The value of the expression $7^{\dfrac{2-\log_{14}7}{\log_{14}49}}$ is equal to:

(A) $2\sqrt{7}$; B) $7\sqrt{2}$; C) $2\sqrt{14}$; D) $\sqrt{7}$; E) $\sqrt{14}$; N) I don't know.

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9. Let x_1 and x_2 be the solutions of the equation $2x^2 + 2mx - m - 4 = 0$. The sum of all integer values of the parameter m for which $x_1 < 0$ and $x_2 > 1$ is:

A) 0; (B) -5; C) -2; D) -3; E) -6; N) I don't know.

10. Consider the equation $6^{3x^2} - 3^{2x^2+1} \cdot 4^{x^2} + 3^{x^2+1} \cdot 2^{x^2} = 126$. The difference between the largest and the smallest solution of this equation is equal to:

A) 12; B) 6; (C) 2; D) 18; E) 4; N) I don't know.

11. The set of all solutions of the inequality $\sqrt{\frac{9}{x^2} - 3} > 1 + \frac{3}{x}$ is a subset of:

A) $[-1, +\infty)$; (B) $(-2, 0)$; C) $(-\sqrt{2}, 2)$; D) $(-\infty, -1]$; E) $(-\sqrt{3}, \sqrt{3})$; N) I don't know.

12. The product of all solutions of the equation $\frac{1}{1 - \log_x 16} + \frac{4}{1 + \log_x 4} = 1$ is equal to:

A) 1; B) 2; C) 16; (D) 8; E) 4; N) I don't know.

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13. If $\cos 2\alpha = \sin \alpha + \frac{5}{8}$ and $\alpha \in \left(\frac{\pi}{2}, \pi\right)$, then the value of $\cos \alpha$ is equal to:

A) $-\frac{2\sqrt{2}}{3}$; B) $-\frac{7}{8}$; C) $-\frac{2\sqrt{6}}{5}$; D) $-\frac{\sqrt{3}}{2}$; (E) $-\frac{\sqrt{15}}{4}$; N) I don't know.

14. Let AA_1 and CC_1 be medians and CC' be the height of triangle ABC. If $\measuredangle BAC = 45°$ and $|AB| = 3 \cdot |CC'|$, then $|AA_1| : |CC_1|$ is equal to:

A) $2 : 1$; B) $\sqrt{17} : 2$; (C) $\sqrt{17} : \sqrt{5}$; D) $3 : 2$; E) $4 : \sqrt{5}$; N) I don't know.

15. For the terms of a geometric progression $a_1, a_2, a_3, \ldots$ whose all terms are positive, we have that $a_3^3 + a_7 = 2a_4^2$ and $a_5 = 2a_2 + 3a_3$. The sum of the first ten terms of this progression is:

A) $\frac{5^{10} - 1}{4}$; B) $\frac{3^{10} - 1}{2}$; C) $\frac{2^{10} - 1}{2}$; (D) $2^{10} - 1$; E) $3^{10} - 1$; N) I don't know.

16. The sum of the radii of all circles that contain the points $A\left(\frac{3}{5}, \frac{6}{5}\right)$ and $B(-1, 2)$ and are tangent to the line $x = 1$ is equal to:

(A) 3; B) $\frac{5}{2}$; C) $\frac{7}{2}$; D) $2\sqrt{3}$; E) $2\sqrt{2}$; N) I don't know.

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17. An isosceles trapezoid with bases of lengths 10 cm and 4 cm is rotated about its midline. If the volume of the resulting solid is $36\pi\,\mathrm{cm}^3$, its surface area is equal to:

(A) $50\pi\,\mathrm{cm}^2$; B) $56\pi\,\mathrm{cm}^2$; C) $45\pi\,\mathrm{cm}^2$; D) $40\pi\,\mathrm{cm}^2$; E) $48\pi\,\mathrm{cm}^2$; N) I don't know.

18. The number of all solutions of the equation $2\tan^2 x \cos x - \tan x + 2\sin x = 1$ in the interval $(-\pi, \pi)$ is equal to:

A) 3; B) 2; C) 1; D) 5; (E) 4; N) I don't know.

19. The sum of the first three binomial coefficients in the expansion of $\left(\frac{\sqrt[3]{y}}{x} - \frac{\sqrt{x}}{y}\right)^n$, $x > 0$, $y \neq 0$ is 121. The term of the expansion that contains x^3 is equal to:

A) $-455x^3y^{-11}$; B) $105x^3y^{-11}$; C) $-1365x^3y^{-12}$; D) $-455x^3y^{-12}$; (E) $455x^3y^{-11}$; N) I don't know.

20. The number of all odd six-digit numbers with distinct digits, in which the digits 1 and 2 are adjacent, is equal to:

A) 7650; B) 7440; C) 7560; (D) 7470; E) 7680; N) I don't know.