University of Belgrade
Faculty of Organizational Sciences

29 June 2021

MATHEMATICS ENTRANCE EXAM

The test contains 20 questions on 2 pages. Each question is worth 5 points. If you do not wish to choose one of the first five offered answers, you may mark "N", which is worth 0 points. For an incorrect answer, 0.5 points are deducted. If, for a given question, more than one answer is marked or no answer is marked, as well as if the answer is marked incorrectly in any way, 1 point is deducted.

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1. If $z = \dfrac{3+i}{4-2i} - \dfrac{1-2i}{3-i}$, $i^2 = -1$, then z^{2021} is equal to:

A) $-\dfrac{1+i}{\sqrt{2}}$; B) 1; C) $-i$; D) $\dfrac{i-1}{\sqrt{2}}$; (E) i; N) I don't know.

2. If $f(x+2) = 2x-1$, $g(f(x)-2) = 2x+1$, and g^{-1} is the inverse function of g, then the value of $g^{-1}(f(0))$ is equal to:

A) -10; B) -3; C) -8; (D) -13; E) -5; N) I don't know.

3. The value of the expression $\left[\left[(0.5)^{-\frac{1}{2}} \cdot 8^{\frac{2}{3}}\right] : \dfrac{2}{3}\right]^2 \cdot 0.125$ is equal to:

A) 11; B) 7; (C) 9; D) 6; E) 12; N) I don't know.

4. For $|a| \neq |b|$, the expression $\left(\dfrac{a^2+b^2}{a^2+ab+b^2} : \dfrac{a^4-b^4}{a^3-b^3}\right)^{-2} : \left(a^2-b^2\right)$ is identically equal to:

A) $\dfrac{2}{a-b}$; B) $a+b$; (C) $\dfrac{a+b}{a-b}$; D) $\dfrac{a-b}{a+b}$; E) $\dfrac{2}{a+b}$; N) I don't know.

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5. From a tank, 20% of the water that was in it at the beginning of the day leaks out each day. If there are 1000 liters of water in the tank, after three days there will be:

A) 500 liters; B) 476 liters; C) 524 liters; D) 488 liters; (E) 512 liters; N) I don't know.

6. The sum of all real solutions of the equation $(x^2+x-12)\sqrt{\sqrt{2x^2+x-9}+x-1} = 0$ is equal to:

A) 1; B) 2; (C) -2; D) -1; E) 0; N) I don't know.

7. The sum of the squares of all solutions of the equation $81^{x} \cdot 3^{x^5-1} = \dfrac{1}{9} \cdot 243^{x^3+\frac{1}{5}}$ is equal to:

(A) 10; B) 13; C) 20; D) 5; E) 26; N) I don't know.

8. The number of all integer values of the parameter m for which the inequality $\dfrac{x-m}{x^2-x+1} > \dfrac{x-2m}{x^2+x+1}$ holds for all real values of x, is equal to:

A) 4; B) 0; (C) 1; D) 2; E) 3; N) I don't know.

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9. The remainder obtained by dividing the polynomial $P(x) = x^{2021} - 20x^3 + 2000x - 20$ by the polynomial $Q(x) = x^3 + x$ is equal to:

A) $x^2+2000x-20$; B) $2020x - 21$; C) $x^2+2021x-20$; D) $x^2+2020x-21$; (E) $2021x - 20$; N) I don't know.

10. The value of the expression $\left(25^{\log_{\sqrt{5}} \sqrt{2}} + 9^{\log_{\frac{1}{3}} \frac{1}{2}}\right)^{\frac{1}{3}}$ is equal to:

A) 4; B) $2\sqrt{2}$; C) $2\sqrt[3]{2}$; (D) 2; E) $\sqrt[3]{4}$; N) I don't know.

11. In triangle ABC, point D is the foot of the perpendicular from vertex C to side AB. If $|AD| = 2\,\text{cm}$, $|CD| = 4\,\text{cm}$, and $|BD| = 8\,\text{cm}$, then the sum of the radii of the circles circumscribed around the triangles ABC, ACD, and BCD is equal to (in cm):

(A) $5 + 3\sqrt{5}$; B) $1 + 5\sqrt{5}$; C) $3 + 4\sqrt{5}$; D) $9 + \sqrt{5}$; E) $7 + 2\sqrt{5}$; N) I don't know.

12. The apex of an isosceles triangle ABC is the point $A(-1, 0)$, and vertices B and C lie on the parabola $y^2 = 4x$. If the point $(0, 0)$ is the orthocenter of triangle ABC, then its area is equal to:

A) 20; (B) $8\sqrt{3}$; C) $\frac{7}{2}\sqrt{10}$; D) $6\sqrt{2}$; E) $\frac{9}{2}\sqrt{14}$; N) I don't know.

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13. The value of the expression $\dfrac{\sin 95^\circ \cos 65^\circ - \sin 5^\circ \sin 65^\circ}{\cos 80^\circ - \cos 40^\circ}$ is equal to:

A) $-\frac{1}{3}$; B) $\frac{\sqrt{3}}{3}$; C) $\frac{\sqrt{2}}{2}$; (D) $-\frac{\sqrt{3}}{3}$; E) $-\frac{\sqrt{2}}{2}$; N) I don't know.

14. Let $a_1, a_2, a_3, \ldots$ be an arithmetic progression with distinct terms. If a_1, a_6 and a_{10} are consecutive terms of some geometric progression, then the value of $\dfrac{a_{2021}}{a_1}$ is equal to:

A) $-\frac{401}{5}$; B) -80; C) $-\frac{403}{5}$; (D) $-\frac{399}{5}$; E) -81; N) I don't know.

15. The product of all real solutions of the equation $1 + \log_{6-x}(6 + x) = 2\log_{6+x}(36 - x^2)$ is equal to:

A) -140; (B) -105; C) -175; D) -70; E) -210; N) I don't know.

16. Three married couples are to be seated in 6 different seats in a single row of a cinema hall with 12 numbered seats. The number of ways to do this so that each couple sits in two adjacent seats is equal to:

(A) 4032; B) 4080; C) 4128; D) 3936; E) 3984; N) I don't know.

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17. The number of all solutions of the equation $\sin 3x \cos 4x + \sin^3 x = 3 \sin x \cos^2 x$ in the interval $(0, 2\pi)$ is equal to:

A) 5; B) 6; C) 4; D) 3; (E) 7; N) I don't know.

18. The maximum volume of a right regular hexagonal prism with surface area P is equal to:

A) $\frac{\sqrt[4]{3}}{9}P\sqrt{P}$; B) $\frac{\sqrt{3}}{18}P\sqrt{P}$; C) $\frac{\sqrt{3}}{9}P\sqrt{P}$; (D) $\frac{\sqrt[4]{3}}{18}P\sqrt{P}$; E) $\frac{1}{18}P\sqrt{P}$; N) I don't know.

19. The sum of all integer solutions of the equation $(x + 2)(x + 3)(x + 8)(x + 12) = 4x^2$ is equal to:

A) 10; B) 0; (C) -10; D) -5; E) 5; N) I don't know.

20. The number of terms of the expansion $\left(\sqrt{2} + \sqrt[3]{6}\right)^n$ that are natural numbers is 35, and the total number of terms is divisible by 6. The number of irrational terms of this expansion is:

A) 181; (B) 175; C) 157; D) 169; E) 163; N) I don't know.