University of Belgrade
Faculty of Organizational Sciences

28 June 2022

MATHEMATICS ENTRANCE EXAM

The test contains 20 questions on 2 pages. Each question is worth 5 points. If you do not wish to choose one of the first five offered answers, you may mark "N", which is worth 0 points. For an incorrect answer, 0.5 points are deducted. If, for a given question, more than one answer is marked or no answer is marked, as well as if the answer is marked incorrectly in any way, 1 point is deducted.

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1. The value of the expression $\left[\left[(0.2)^{-1}\cdot 2^{0.5}\right] : \frac{5}{3}\right]^2 \cdot (-3)^{-2}$ is equal to:

A) 4; B) 3; C) 5; (D) 2; E) 1; N) I don't know.

2. For $a \in (0,4)$, the expression $\sqrt{\frac{1}{4}a + 4a^{-1} - 2} + \sqrt{\frac{1}{4}a + \frac{1}{4}a^{-1} + \frac{1}{2}}$ is identically equal to:

A) $\frac{7}{2\sqrt{a}}$; B) $\frac{3}{\sqrt{a}}$; (C) $\frac{5}{2\sqrt{a}}$; D) $\frac{3}{2\sqrt{a}}$; E) $\frac{2}{\sqrt{a}}$; N) I don't know.

3. If $f\left(\frac{x+1}{x-1}\right) = x$ for $x \neq 1$, $g(x) = \frac{2}{f(x)-1}$ and g^{-1} is the inverse function of g, then the value of $f(g^{-1}(1))$ is equal to:

A) $\frac{1}{3}$; B) $\frac{1}{2}$; C) 2; D) -1; (E) 3; N) I don't know.

4. If $z = \left(5 - \frac{3+7i}{1+i}\right)^{2022}$, $i^2 = -1$, then $z + \bar{z}$ is equal to:

(A) -2^{2023}; B) 2^{2023}; C) -2^{2022}; D) 2^{2022}; E) -2^{2021}; N) I don't know.

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5. If, by increasing the length of one side of a square by 25% and the other by p%, we obtain a rectangle whose area is twice the area of the square, then:

A) $p = 75$; (B) $p = 60$; C) $p = 65$; D) $p = 50$; E) $p = 70$; N) I don't know.

6. For the terms of an increasing geometric progression $a_1, a_2, a_3, \ldots$ the equalities $a_1^3 = 2a_2$ and $\frac{a_4}{a_2} - \frac{a_2}{a_1} = 2$ hold. The product of the first five terms of the given sequence is equal to:

A) 2^{10}; (B) 2^{15}; C) 2^5; D) 2^{20}; E) 2^{25}; N) I don't know.

7. The remainder obtained by dividing the polynomial $P(x) = x^{2022} - 2x^{2021} + 4$ by the polynomial $Q(x) = x^2 - x - 2$ is equal to:

A) $-2x + 5$; B) $-3x + 4$; C) $-5x + 2$; (D) $-x + 6$; E) $-4x + 3$; N) I don't know.

8. The value of the expression $\left(\log_{\sqrt[3]{3}} \sqrt{3}\right)^2 \cdot \log_{(\sqrt{3}+1)}(4 + 2\sqrt{3}) + \log_{\sqrt[3]{2}} 2\sqrt{2}$ is equal to:

A) 12; B) 8; (C) 9; D) 10; E) 15; N) I don't know.

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9. The number of all integer solutions of the inequality $\dfrac{11-16x}{4x^2-8x-21} \geq 1$ is equal to:

A) 1; (B) 5; C) 3; D) 4; E) 2; N) I don't know.

10. The product of all real solutions of the equation $28 \cdot \sqrt{3}^{x^2-x} - 3^{x^2-x+2} = 3$ is equal to:

A) -10; B) -6; (C) -2; D) -4; E) -8; N) I don't know.

11. The number of all solutions of the equation $2\left(\cos^3 x + \sin^2 \dfrac{x}{2}\right) = 1 + \cos 2x$ in the interval $(-\pi, \pi)$ is equal to:

(A) 5; B) 4; C) 6; D) 2; E) 3; N) I don't know.

12. The product of all real solutions of the equation $\log_3 x \cdot \log_3 9x = \log_3 3x$ is equal to:

A) $\dfrac{1}{6}$; B) 3; C) 1; D) 9; (E) $\dfrac{1}{3}$; N) I don't know.

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13. The value of the expression $\dfrac{\cos 10^\circ - \sin 10^\circ}{\cos 95^\circ + \cos 205^\circ}$ is equal to:

A) $-\dfrac{2}{3}$; B) $-\dfrac{2}{\sqrt{3}}$; C) $-\dfrac{\sqrt{2}}{3}$; D) -1; (E) $-\dfrac{\sqrt{2}}{\sqrt{3}}$; N) I don't know.

14. Let B and C be the points of tangency of the tangents from the point $A(6,0)$ to the circle $(x-1)^2 + y^2 = 9$. The length of the segment BC is equal to:

A) $\dfrac{23}{5}$; (B) $\dfrac{24}{5}$; C) 5; D) $\dfrac{22}{5}$; E) 4; N) I don't know.

15. The sum of the squares of all integer solutions of the inequality $\sqrt{4x^2-4x+1} + 2\sqrt{-2x^2+10x-8} \leq 4x-3$ is equal to:

A) 30; B) 21; C) 17; D) 10; (E) 26; N) I don't know.

16. From a square $ABCD$ with side length 2 cm, two triangles are cut: one with vertices at point B and the midpoints of sides AB and BC, and the other with vertices at point D and the midpoints of sides CD and DA. The volume of the solid obtained by rotating the remaining part of the square about the line containing side AB is equal to:

A) $4\pi\,\mathrm{cm}^3$; B) $5\pi\,\mathrm{cm}^3$; (C) $6\pi\,\mathrm{cm}^3$; D) $7\pi\,\mathrm{cm}^3$; E) $8\pi\,\mathrm{cm}^3$; N) I don't know.

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17. The product of the smallest and the largest value of the function $f(x) = 8^x - \dfrac{3}{2}16^x$ on the interval $[-4, 4]$ is equal to:

(A) $-23 \cdot 2^7$; B) $-3 \cdot 2^{10}$; C) $-5 \cdot 2^9$; D) $-21 \cdot 2^7$; E) $-25 \cdot 2^7$; N) I don't know.

18. The number of terms in the expansion of $\left(\sqrt[6]{54} + \sqrt[3]{32}\right)^{2022}$ that are integers is equal to:

A) 337; B) 674; C) 675; D) 338; (E) 1012; N) I don't know.

19. In an acute triangle ABC, the bisector of the interior angle at vertex A intersects side BC at point D. If $|AB| = |AD| = |DC| = 2\,\mathrm{cm}$, then the perimeter of the given triangle (in cm) is equal to:

A) 8; B) $6 + \sqrt{5}$; C) $4\sqrt{5}$; D) 9; (E) $4 + 2\sqrt{5}$; N) I don't know.

20. The number of ways to arrange 4 boys and 6 girls in a row, such that no two boys are next to each other, is equal to:

A) $140 \cdot 7!$; B) $160 \cdot 7!$; C) $150 \cdot 7!$; (D) $120 \cdot 7!$; E) $180 \cdot 7!$; N) I don't know.