University of Belgrade
Faculty of Organizational Sciences

27 June 2023

MATHEMATICS ENTRANCE EXAM

The test contains 20 questions on 2 pages. Each question is worth 5 points. If you do not wish to choose one of the first five offered answers, you may mark "N", which is worth 0 points. For an incorrect answer, 0.5 points are deducted. If, for a given question, more than one answer is marked or no answer is marked, as well as if the answer is marked incorrectly in any way, 1 point is deducted.

---

1. The value of the expression $\left((0.2)^{-2} + \sqrt[3]{64} \cdot \left(13^2 - 12^2\right)\right)^{\frac{1}{3}} : \sqrt[3]{(-2)^3}$ is equal to:

A) $\frac{5}{2}$; B) $\frac{1}{2}$; C) $-\frac{1}{2}$; D) $-\frac{3}{2}$; (E) $-\frac{5}{2}$; N) I don't know.

2. For $b \neq 0$, the expression $\left(\frac{a^3}{b^3} + 1\right) : \left(\frac{a^2}{b^2} - \frac{a}{b} + 1\right)$ is identically equal to:

(A) $\frac{a+b}{b}$; B) $\frac{a+3b}{2b}$; C) $\frac{2b}{a}$; D) $\frac{2a}{b}$; E) $\frac{3a+b}{2b}$; N) I don't know.

3. If $f(x) = \frac{x}{x+5}$ for $x \neq -5$, $g(x) = \frac{5}{5-x}$ for $x \neq 5$, and $h(x) = f^{-1}(x) \cdot g^{-1}(x)$ for $x \neq 0$ and $x \neq 1$, where f^{-1} and g^{-1} are the corresponding inverse functions, then:

A) $h(x) = -1$; B) $h(x) = 1$; C) $h(x) = 5$; D) $h(x) = -5$; (E) $h(x) = -25$; N) I don't know.

4. If $z^2 - |z|^2 + 4 \cdot \operatorname{Im} z = 2 - 6i$, $i^2 = -1$, then $z \cdot \bar{z}$ is equal to:

A) 5; (B) 10; C) 1; D) 2; E) 17; N) I don't know.

---

5. The price of a book is first reduced by 10%, and then increased by 900 dinars. If the new price is 50% greater than the old price, then the new price of the book is:

A) 2400 dinars; B) 1750 dinars; C) 1800 dinars; (D) 2250 dinars; E) 2000 dinars; N) I don't know.

6. For the terms of an arithmetic sequence $a_1, a_2, a_3, \ldots$ the equality $a_4 + a_5 + a_{11} + a_{12} = 32$ holds. The sum of the first 15 terms of this sequence is equal to:

A) 128; B) 144; C) 64; D) 96; (E) 120; N) I don't know.

7. The product of all real solutions of the equation $\left(\log_{\frac{1}{x}} 4\right)^{-2} + 0.5 = 3 \log_{16} x$ is equal to:

A) 64; B) 4; (C) 8; D) 32; E) 16; N) I don't know.

8. The value of the expression $\sqrt[4]{4^{6 \log_8 5 - \log_{\sqrt{2}} 125}}$ is equal to:

A) $\frac{1}{4}$; B) $\frac{1}{9}$; C) $\frac{1}{36}$; (D) $\frac{1}{25}$; E) $\frac{1}{16}$; N) I don't know.

---

9. The sum of all integer solutions of the inequality $\dfrac{8x-3}{(x+1)^2(x+3)(x-2)} \geq \dfrac{1}{(x+1)(x-2)}$ is equal to:

A) 1; B) 0; C) -3; (D) -1; E) 3; N) I don't know.

10. The sum of squares of all real solutions of the equation $2\sqrt{2}\left(1+\sqrt{2}\right)^{x+1} - \left(3+2\sqrt{2}\right)^{x+1} = 1$ is equal to:

(A) 4; B) 1; C) 9; D) 8; E) 5; N) I don't know.

11. The number of all real solutions of the equation $(\sqrt{3}-1)\sin x + \sqrt{3}\cos x = \sin x \tan x$ in the interval $\left(-\pi, \dfrac{3\pi}{2}\right]$ is equal to:

A) 4; (B) 5; C) 1; D) 2; E) 3; N) I don't know.

12. The remainder obtained by dividing the polynomial $P(x) = (x-1)^{2023} + x^3 + 1$ by the polynomial $Q(x) = x(x^2 - 2x + 2)$ is equal to:

A) $2x^2 + x$; B) $x^2 + x$; C) $2x^2 - x$; (D) $x^2 - x$; E) $3x^2 - x$; N) I don't know.

13. The value of the expression $\dfrac{4\sin 50^\circ \sin 185^\circ + \sqrt{2}}{\sin 10^\circ - \cos 10^\circ}$ is equal to:

A) 2; B) -2; (C) $-\sqrt{2}$; D) 1; E) -1; N) I don't know.

14. The sum of all values of the real parameter p for which the line $y = 2x + p$ is tangent to the circle $x^2 + 2x + y^2 - 4y = 10$ is equal to:

(A) 8; B) 10; C) 9; D) 12; E) 6; N) I don't know.

15. The difference between the largest and the smallest solution of the inequality $x\sqrt{x^2+x-6} \geq 2x^2 - 4x$ is equal to:

A) $\dfrac{14}{3}$; B) $\dfrac{2}{3}$; C) $\dfrac{11}{3}$; (D) $\dfrac{5}{3}$; E) $\dfrac{8}{3}$; N) I don't know.

16. If the length of the height of a right regular hexagonal pyramid is three times the length of the side of its base, then the ratio of the lateral surface area to the base area of that pyramid is equal to:

A) $2\sqrt{3} : 1$; (B) $\sqrt{13} : 1$; C) $2\sqrt{11} : \sqrt{3}$; D) $3\sqrt{2} : 1$; E) $2\sqrt{10} : \sqrt{3}$; N) I don't know.

17. The minimum sum of distances from a point on the x-axis to the points $A(-6, 1)$ and $B(6, 4)$ is equal to:

A) $\dfrac{29}{2}$; (B) 13; C) $\dfrac{25}{2}$; D) $\dfrac{27}{2}$; E) 14; N) I don't know.

18. The product of the third term from the beginning and the third term from the end of the expansion of $\left(\sqrt[n]{2023} + \dfrac{1}{\sqrt[n]{2023}}\right)^n$ is equal to 66^2. The sum of the binomial coefficients in the given expansion is equal to:

A) 128^2; B) 32^2; (C) 64^2; D) 256^2; E) 16^2; N) I don't know.

19. On the sides AB, BC and DA of the square $ABCD$, points M, N and P are given respectively such that $AM : MB = 2 : 1$, $BN : NC = 3 : 2$ and $DP : PA = 4 : 3$. If the side length of the square is $1\,cm$, then the area of triangle MNP is equal to:

(A) $\dfrac{19}{70}\,cm^2$; B) $\dfrac{2}{7}\,cm^2$; C) $\dfrac{3}{10}\,cm^2$; D) $\dfrac{9}{35}\,cm^2$; E) $\dfrac{11}{35}\,cm^2$; N) I don't know.

20. In a tennis match, Djokovic defeated Nadal in two sets, with the scores $6 : 3$ and $6 : 4$ in games (a set is won by the player who first wins 6 games in that set). The number of different ways the game-by-game score of this match could have progressed is equal to:

A) 72^2; B) 96^2; C) 90^2; D) 78^2; (E) 84^2; N) I don't know.