University of Belgrade
Faculty of Organizational Sciences

25 June 2024

MATHEMATICS ENTRANCE EXAM

The test contains 20 questions on 2 pages. Each question is worth 5 points. If you do not wish to choose one of the first five offered answers, you may mark "N", which is worth 0 points. For an incorrect answer, 0.5 points are deducted. If, for a given question, more than one answer is marked or no answer is marked, as well as if the answer is marked incorrectly in any way, 1 point is deducted.

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1. Masha received 13000 dinars from her parents, which represents 26% of the price of the laptop she wants to buy. If she received an additional 13500 dinars from her grandmother, then the amount Masha still needs to buy the laptop is equal to:

A) 24000 dinars; B) 22500 dinars; C) 24200 dinars; (D) 23500 dinars; E) 22800 dinars; N) I don't know.

2. The value of the expression $\left(36.5+\left(\frac{1}{25}\right)^{-\frac{1}{2}}-\frac{3}{2}\right)^{\frac{1}{2}}\cdot\left(\sqrt[3]{27}+2^3-1\right)^{-\frac{1}{2}}$ is equal to:

A) $2\sqrt{10}$; B) 20; C) 10; (D) 2; E) 1; N) I don't know.

3. For $a\neq 0$, $b\neq 0$ and $|a|\neq|b|$, the expression $\left(\frac{ab}{a-b}+a\right)\cdot\left(\frac{ab}{a+b}-a\right):\frac{a^2b^2}{a^2-b^2}$ is identically equal to the expression:

(A) $-\frac{a^2}{b^2}$; B) $\frac{1}{a^2b^2}$; C) $\frac{b^2}{a^2}$; D) $-ab$; E) $-\frac{1}{b^2}$; N) I don't know.

4. If $z=\frac{(1-i)^{2025}}{(1+i)^{2024}}$, $i^2=-1$, then $\mathrm{Re}z$ is equal to:

A) 0; B) $\frac{\sqrt{2}}{2}$; (C) 1; D) 2; E) $\sqrt{2}$; N) I don't know.

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5. Let $f(x+3)=2x+5$, $g(f(x)-1)=3x+2$ and $h(x)=3f(x)+2g(x)$. Then:

(A) $h(x)=9x+7$; B) $h(x)=5-7x$; C) $h(x)=5x+9$; D) $h(x)=7x-9$; E) $h(x)=5x-7$; N) I don't know.

6. The sum of all real solutions of the equation $1+9^{\sqrt{x^2+x}+0.5}=4\cdot 3^{\sqrt{x^2+x}}$ is equal to:

(A) -1; B) 0; C) -2; D) 3; E) 2; N) I don't know.

7. The value of the expression $3^{\log_{\sqrt{5}}7\cdot\log_9 25}$ is equal to:

A) 56; B) 48; C) 36; D) 25; (E) 49; N) I don't know.

8. The number of all integer solutions of the inequality $\frac{x^2-12x+20}{x^2-7x+6}\geq 2$ is:

A) 4; B) 2; C) 6; (D) 5; E) 3; N) I don't know.

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9. The sum of the first two terms of a geometric progression is equal to 16, and the difference between the third and the first term is 32. The sum of the first four terms of that sequence is equal to:

A) 192; B) 128; C) 144; D) 176; (E) 160; N) I don't know.

10. All real solutions of the equation $\log_{x-1}(x+1) + 4\log_{x+1}(x-1) = 4$ belong to the interval:

A) $(6, 8]$; (B) $(2, 4]$; C) $(1, 2]$; D) $(4, 6]$; E) $(8, 10]$; N) I don't know.

11. The remainder obtained by dividing the polynomial $P(x) = x^{2024} + x^{25} - x^6 + 1$ by the polynomial $Q(x) = x^3 + x$ is equal to:

A) $3x^2 + 1$; (B) $-2x^2 + x + 1$; C) $x^2 + 2x + 1$; D) $-2x^2 + 2x + 1$; E) $x^2 + 3x + 1$; N) I don't know.

12. The value of the expression $\dfrac{1}{\sin 10^\circ} - \dfrac{\sqrt{3}}{\cos 10^\circ}$ is equal to:

A) $2\sqrt{3}$; B) 3; (C) 4; D) 2; E) $3\sqrt{2}$; N) I don't know.

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13. The number of all integer solutions of the inequality $1 - x < \sqrt{6 + x - x^2}$ is equal to:

A) 1; B) 3; (C) 4; D) 5; E) 6; N) I don't know.

14. If A and B are the contact points of the hyperbola $\dfrac{x^2}{4} - \dfrac{y^2}{3} = 1$ and the circle $x^2 + (y-7)^2 = 32$, and C is the intersection point of their common tangents at points A and B, then the area of the triangle ABC is equal to:

A) $8\sqrt{2}$; B) 14; C) 8; D) 12; (E) 16; N) I don't know.

15. Consider a cylinder and a regular tetrahedron of equal volume. If the length of the base radius of the cylinder is equal to the length of the radius of the circle inscribed in one of the faces of the tetrahedron, then the ratio of the height of the cylinder to the height of the tetrahedron is equal to:

A) $2\sqrt{3} : 3\pi$; (B) $\sqrt{3} : \pi$; C) $\sqrt{2} : \pi$; D) $\sqrt{6} : 2\pi$; E) $\sqrt{3} : 3\pi$; N) I don't know.

16. The minimum lateral surface area of a right circular cone of volume V is equal to:

A) $3 \cdot \sqrt[3]{V^2\pi}$; (B) $3 \cdot \sqrt[3]{\dfrac{\sqrt{3}V^2\pi}{2}}$; C) $3 \cdot \sqrt[3]{\dfrac{3V^2\pi}{4}}$; D) $3 \cdot \sqrt[3]{\dfrac{V^2\pi}{2}}$; E) $3 \cdot \sqrt[3]{\dfrac{\sqrt{3}V^2\pi}{4}}$; N) I don't know.

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17. Let ABC be a right-angled triangle with the right angle at vertex A. Points D and E are given on the segments AB and BC, respectively, such that $|AC| = |AE| = 4$ cm and $|DE| = |DB| = 3$ cm. The area of the triangle BED is equal to:

(A) 3.6 cm^2; B) 4.2 cm^2; C) 2.8 cm^2; D) 3.2 cm^2; E) 4 cm^2; N) I don't know.

18. The number of distinct triangles whose vertices $A_k(x_k, y_k)$, $k \in \{1, 2, 3\}$, have integer coordinates from the set $\{1, 2, 3, 4\}$ is equal to:

A) 524; B) 540; C) 532; D) 510; (E) 516; N) I don't know.

19. The number of terms in the expansion of $\left(\sqrt{6} + \dfrac{1}{\sqrt[3]{2}}\right)^{40}$ that are natural numbers is equal to:

A) 3; B) 2; C) 6; D) 4; (E) 5; N) I don't know.

20. The sum of all real solutions of the equation $11\cos 2x - 3 = 3\sin 3x - 11\sin x$ in the interval $[0, 2\pi)$ is equal to:

A) 3π; B) 2π; C) $\dfrac{7\pi}{3}$; (D) $\dfrac{7\pi}{2}$; E) $\dfrac{5\pi}{6}$; N) I don't know.