University of Belgrade
Faculty of Organizational Sciences

25 July 2025

MATHEMATICS ENTRANCE EXAM

The test contains 20 questions on 2 pages. Each question is worth 5 points. If you do not wish to choose one of the first five offered answers, you may mark "N", which is worth 0 points. For an incorrect answer, 0.5 points are deducted. If, for a given question, more than one answer is marked or no answer is marked, as well as if the answer is marked incorrectly in any way, 1 point is deducted.

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1. The value of the expression $\dfrac{\sqrt{(-2025)^2}+|-2025|+\sqrt[3]{(-2025)^3}}{\sqrt[7]{(-5)^7}+\sqrt[10]{(-6)^{10}}}$ is equal to:

A) $-\dfrac{6075}{11}$; B) -4050; C) 4050; D) $\dfrac{6075}{11}$; (E) 2025; N) I don't know.

2. For $a \neq 0, b \neq 0$ and $a \neq b$, the expression $\left(\left(\dfrac{a^2}{b}+\dfrac{b^2}{a}+3\cdot\dfrac{b^2-a^2}{b-a}\right):(ab)^{-1}\right)^{\frac{1}{3}}$ is identically equal to the expression:

A) $a-b$; B) b; (C) $a+b$; D) $b-a$; E) a; N) I don't know.

3. If $f\left(\dfrac{x-1}{x-2}\right)=x$ for $x\neq 2$, $g(f(x)-2)=x-3$ for $x\neq 1$, $h(x)=\dfrac{1}{2x}$ for $x\neq 0$ and g^{-1} is the inverse function of function g, then the value of the expression $f(0)+g^{-1}(-3)+h(h(h(1/4)))$ is equal to:

A) $\dfrac{3}{4}$; B) 4; C) $\dfrac{1}{4}$; D) 0; (E) 2; N) I don't know.

4. If $z=\left(1-\dfrac{1-i}{1+i}\right)^{2025}$, $i^2=-1$, then $\mathrm{Re}(z)$ is equal to:

A) 2^{2023}; (B) 2^{1012}; C) 2^{2024}; D) 2^{2013}; E) 2^{2025}; N) I don't know.

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5. The initial prices of a mathematics book and a physics book are equal. If the price of the mathematics book first increases by 20%, and then by an additional 960 dinars, and the price of the physics book increases by 52%, their new prices will again be equal. The initial price of the mathematics book is:

A) 3500 dinars; B) 3100 dinars; C) 3200 dinars; (D) 3000 dinars; E) 2800 dinars; N) I don't know.

6. If a_1, a_2 and a_3 are the first three terms of a geometric progression whose common ratio q is a natural number and if $a_1=8$ and $2a_2-\dfrac{a_3}{2}>15$, then $a_1+a_2+a_3$ is equal to:

A) 128; B) 52; C) 64; D) 60; (E) 56; N) I don't know.

7. The product of all real solutions of the equation $x^{2+\log_3 x}=3^8$ is equal to:

(A) $\dfrac{1}{9}$; B) 3; C) $\dfrac{1}{3}$; D) $\dfrac{1}{81}$; E) 81; N) I don't know.

8. The value of the expression $\log_2\left(3^{(2-\log_3 72)}\cdot 5^{(\log_5(4/5)+1)}\right)$ is equal to:

A) 1; B) -3; C) -2; (D) -1; E) 2; N) I don't know.

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9. The sum of all integer solutions of the inequality $\dfrac{x^2+x-15}{x^2-2x-3} \geq 2$ is equal to:

A) 1; B) 5; C) 4; (D) 3; E) 2; N) I don't know.

10. The product of all real solutions of the equation $4^{\sqrt{x^2-3}} + 8 = 3 \cdot 2^{1+\sqrt{x^2-3}}$ is equal to:

(A) 28; B) 7; C) -14; D) 4; E) 14; N) I don't know.

11. The number of all solutions of the equation $3\sin x + (\cos 2x - 1)\cot^2 x = 0$ belonging to the interval $[0, 2\pi)$ is equal to:

A) 4; (B) 2; C) 1; D) 5; E) 3; N) I don't know.

12. The remainder obtained by dividing the polynomial $P(x) = x^{2025} + x^{2024} - x^{2023} + 1$ by $Q(x) = x^3 + x^2 + x + 1$ is equal to:

(A) $x^2 + 2x + 3$; B) $2x + 1$; C) $3^2 + 2x + 1$; D) $4x^2 + 2x - 1$; E) $x^2 + 3x + 2$; N) I don't know.

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13. If $\sin\alpha = \dfrac{4}{5}$ and $\dfrac{\pi}{2} < \alpha < 2\pi$, then $\tan\left(\dfrac{3\pi}{4} - 2\alpha\right)$ is equal to:

A) $-\dfrac{17}{31}$; B) $-\dfrac{31}{17}$; C) $\dfrac{17}{31}$; D) 1; (E) $\dfrac{31}{17}$; N) I don't know.

14. Let t be the tangent to the circle $(x-3)^2 + (y+2)^2 = 5$ at the point $T(4,0)$. If $M(a,b)$ is the point symmetric to the origin with respect to the tangent t, then the value of the expression $a+2b$ is equal to:

A) $\dfrac{42}{5}$; B) $\dfrac{38}{5}$; (C) 8; D) $\dfrac{36}{5}$; E) $\dfrac{32}{5}$; N) I don't know.

15. The number of integer solutions of the inequality $3x + 4\sqrt{5^4 - x^2} > 0$ is equal to:

A) 21; B) 15; C) 40; (D) 45; E) 50; N) I don't know.

16. If a cone is inscribed in a sphere of radius R such that the lateral surface area of the cone is twice the area of its base, then the volume of that cone is equal to:

A) $\dfrac{1}{2}R^3\pi$; B) $\dfrac{2\sqrt{3}}{9}R^3\pi$; C) $\dfrac{\sqrt{3}}{4}R^3\pi$; D) $\dfrac{1}{3}R^3\pi$; (E) $\dfrac{3}{8}R^3\pi$; N) I don't know.

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17. The value of the real parameter a for which the functions $f(x) = ax^2 - ax + a - 1$ and $g(x) = 3ax^2 - 2x + a$ have equal (finite) maximum values is:

A) $-2 + \dfrac{4}{\sqrt{3}}$; (B) $-2 - \dfrac{4}{\sqrt{3}}$; C) $-4 + \dfrac{2}{\sqrt{3}}$; D) $-4 + \dfrac{\sqrt{3}}{3}$; E) $-4 - \dfrac{\sqrt{3}}{3}$; N) I don't know.

18. The sum of all binomial coefficients in the expansion of $\left(\dfrac{1}{\sqrt{401}} + \sqrt[3]{802}\right)^n$ is equal to 8^{100}. The number of terms in this expansion that are not integers is equal to:

A) 270; B) 250; (C) 280; D) 290; E) 260; N) I don't know.

19. A circle is circumscribed about an isosceles trapezoid $ABCD$ with bases $|AB| = 10$ cm and $|CD| = 6$ cm, such that its center lies on the larger base. The length of the radius of the circle inscribed in triangle ABC is equal to (in cm):

(A) $3\sqrt{5} - 5$; B) $5\sqrt{5} - 9$; C) $2\sqrt{5} - 3$; D) $4\sqrt{5} - 7$; E) $\sqrt{5} - 1$; N) I don't know.

20. In a hospital, there are 6 general practitioners, 4 nurses, and 2 surgeons. It is necessary to select a six-member team for field duty such that the team includes at least two general practitioners and at least one nurse. The number of ways such a team can be selected is equal to:

A) 849; B) 879; C) 869; D) 5040; (E) 859; N) I don't know.