

MOCK MATHEMATICS ENTRANCE EXAM

The test contains 20 questions on 2 pages. Each question is worth 5 points. If you do not wish to choose one of the first five offered answers, you may mark “N”, which is worth 0 points. For an incorrect answer, 0.5 points are deducted. If, for a given question, more than one answer is marked or no answer is marked, as well as if the answer is marked incorrectly in any way, 1 point is deducted.

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1. The value of the expression $\frac{\sqrt{27} - \sqrt{12}}{\sqrt{3}} + \frac{1}{2 - \sqrt{5}} + \frac{1}{2 + \sqrt{5}}$ is equal to:
A) 3; B) $\sqrt{5} + \frac{1}{4}$; C) -3; D) $\sqrt{5} + 4$; E) $\sqrt{5} - 4$; N) I don't know.
2. The price of a product was first increased by 20%, and then decreased by 25%. The new price, compared to the initial one, is:
 A) less by 10%; B) less by 5%; C) greater by 5%; D) greater by 10%; E) unchanged; N) I don't know.
3. If $z = \frac{10i + 5}{1 + 3i}$, where $i^2 = -1$, then the value of $|z|$ is equal to:
 A) $\frac{5}{\sqrt{2}}$; B) $\sqrt{3}$; C) $2\sqrt{3}$; D) 2; E) $\frac{\sqrt{13}}{\sqrt{2}}$; N) I don't know.
4. If $|x| \neq |y|$, then the expression $\frac{x^4 + y^4 - 2x^2y^2}{4 \cdot (x^4 - y^4)} \cdot \left(\frac{x^2 + xy}{x - y} - \frac{x^2 - xy}{x + y} \right)$ is identically equal to:
A) $\frac{x^2y}{x^2 - y^2}$; B) $\frac{xy^2}{x^2 + y^2}$; C) $\frac{x^2y^2}{x^2 - y^2}$; D) $\frac{x^2y}{x^2 + y^2}$; E) $\frac{xy^2}{x^2 - y^2}$; N) I don't know.
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5. If, for $x \neq -2$, $f\left(\frac{2-x}{2+x}\right) = x^{2026} + x^{1013} + 1$, then the value of $f(f(3))$ is equal to:
A) 3; B) 1; C) $\frac{1}{3}$; D) $\frac{1}{2}$; E) 2; N) I don't know.
6. The number of all integer solutions of the inequality $\frac{(x-5)^2}{(2x-3)^2} > 1$ is equal to:
 A) 4; B) 2; C) 1; D) 3; E) 0; N) I don't know.
7. The value of the expression $5^{\log_{\sqrt{5}} 2 + \log_{25} 9}$ is equal to:
A) 5; B) 18; C) 4; D) 12; E) 6; N) I don't know.
8. If the points $A(1, -3)$ and $B(-2, 0)$ belong to the circle k whose center lies on the line $x + y = 0$, then the length of the radius of the circle k is equal to:
A) $\frac{\sqrt{13}}{6}$; B) 6; C) $\frac{\sqrt{13}}{\sqrt{2}}$; D) $\frac{\sqrt{13}}{2}$; E) $\sqrt{6}$; N) I don't know.
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9. The number of all real solutions of the equation $\left(9 \cdot 3^{\frac{x-2}{x+1}}\right)^{\frac{x^2-1}{3x}} \cdot \frac{1}{\sqrt{x+1}} = 3^{x-\sqrt{x}-4}$ is equal to:
- A) 1; B) 2; C) 4; D) 5; E) 3; N) I don't know.
10. If the sum of the first and fourth terms of an increasing geometric progression is equal to 45, and the sum of the second and third terms is equal to 30, then the fifth term of that progression is equal to:
- A) 45; B) 20; C) 80; D) 40; E) $\frac{5}{2}$; N) I don't know.
11. The number of all integer solutions of the inequality $x + 5 < \sqrt{x + 47}$ is equal to:
- A) 46; B) 14; C) 48; D) 7; E) 49; N) I don't know.
12. The number of all integer solutions of the inequality $\log_{0.5}(x+1) + \log_2 \frac{1}{x+5} > -5$ is equal to:
- A) 4; B) 1; C) 0; D) 2; E) 3; N) I don't know.
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13. If, when dividing the polynomial $P(x) = x^4 - x^3 + ax^2 + bx + 4$, $a, b \in \mathbb{R}$, by the polynomial $x^2 + 4$, the remainder obtained is equal to $-x$, then $a + b$ is equal to:
- A) 8; B) 5; C) 0; D) -3 ; E) 2; N) I don't know.
14. The value of the expression $\cot 81^\circ - \cot 27^\circ - \cot 63^\circ + \cot 9^\circ$ is equal to:
- A) 1; B) 4; C) 3; D) 5; E) 2; N) I don't know.
15. Let two concentric circles be given such that a chord of the larger circle, of length 10 cm, is also a tangent to the smaller circle. Then the area of the circular ring formed by these two circles is equal to:
- A) $5\pi \text{ cm}^2$; B) $30\pi \text{ cm}^2$; C) $25\pi \text{ cm}^2$; D) $8\pi \text{ cm}^2$; E) $20\pi \text{ cm}^2$; N) I don't know.
16. Four corner squares with a side length of 1 cm are cut out and removed from a rectangle with side lengths of 4 cm and 3 cm. Each vertex of the rectangle is also a vertex of exactly one removed square, and two sides of that square lie on the sides of the rectangle that intersect at that vertex. The resulting figure is rotated around the line determined by the shorter side of the original rectangle. The volume of the resulting solid of revolution is equal to:
- A) $32\pi \text{ cm}^3$; B) $16\pi \text{ cm}^3$; C) $24\pi \text{ cm}^3$; D) $20\pi \text{ cm}^3$; E) $18\pi \text{ cm}^3$; N) I don't know.
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17. The sum of all solutions of the equation $\frac{(\tan^2 x + \sqrt{3}) \cos^2 x}{\sin 2x} = \frac{1 + \sqrt{3}}{2}$ which are elements of the interval $[0, 2\pi]$ is equal to:
- A) 4π ; B) 3π ; C) $\frac{10\pi}{3}$; D) $\frac{19\pi}{6}$; E) $\frac{18\pi}{4}$; N) I don't know.
18. A tourist plans to visit 7 different cities, including Belgrade, Moscow, and Athens, where each city will be visited exactly once. The number of ways the planned 7 cities can be visited such that Belgrade is visited before Moscow, and Moscow before Athens, is equal to:
- A) 1140; B) 1680; C) 1440; D) 720; E) 840; N) I don't know.
19. The maximal value of the function $f(x) = 3^{x-1} - 27^{x+1}$ is equal to:
- A) $\frac{2\sqrt{3}}{3^3}$; B) $\frac{2\sqrt{3}}{3^5}$; C) $\frac{26\sqrt{3}}{3^8}$; D) $\frac{8}{3^6}$; E) 1; N) I don't know.
20. In the expansion of $\left(\sqrt[5]{x} - \frac{1}{x\sqrt{2}}\right)^{30}$, $x \in \mathbb{R} \setminus \{0\}$, the term that does not contain x is equal to:
- A) $-\frac{25 \cdot 29 \cdot 91}{3\sqrt{2}}$; B) $\frac{23 \cdot 29 \cdot 91}{2}$; C) $-\frac{25 \cdot 29 \cdot 91}{2}$; D) $\frac{27 \cdot 29 \cdot 91}{4\sqrt{2}}$; E) $-\frac{27 \cdot 29 \cdot 91}{2\sqrt{2}}$; N) I don't know.
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